Dr. Lukas Lewark
ETH Zürich

Algebraic Topology II

Problem Sheet 6
Spring, 2024

Manifolds and Orientations

Problem 1. Given the sequence of groups $\mathbb{Z} \xrightarrow{\times 2} \mathbb{Z} \xrightarrow{\times 2} \mathbb{Z} \xrightarrow{\times 2} \cdots$ where each map is multiplication by 2, describe the direct limit of this sequence.

Problem 2. Let X be the union of a directed set of subspaces U_α with the property that each compact set in X is contained in some U_α. Prove that the natural map

$$\varinjlim H_i(U_\alpha; G) \to H_i(X; G)$$

is an isomorphism for all i and G.

Problem 3. Show that $H_c^0(X; G) \cong 0$ if X is path-connected and noncompact.

Problem 4. Compute $H_c^\bullet(X; \mathbb{Z})$, where X is **a)** $\mathbb{R}^n \setminus \{0\}$; **b*)** the open Möbius band.

Problem 5. Let $\varphi\colon S^n \hookrightarrow S^{n+1}$ be a continuous injective map. Prove that the complement has two connected components.