D-MATH | Number Theory II | FS 2024
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Exercise sheet 19

Extensions of Absolute Values, Local and Global Fields

1. Consider a Dedekind ring A with a maximal ideal $\mathfrak{p}$. Let L be a finite Galois extension of $K := \mathrm{Quot}(A)$ with Galois group Γ. Let B be the integral closure of A in L and let $\mathfrak{q}$ be a prime of B above $\mathfrak{p}$. Let K' be the intermediate field corresponding to the decomposition group $\Gamma_{\mathfrak{q}}$, and consider the prime ideal $\mathfrak{p}' := \mathfrak{q} \cap K'$ of $A' := B \cap K'$. Prove that the inclusion $A \hookrightarrow A'$ induces an isomorphism of completions $A_{\mathfrak{p}} \xrightarrow{\sim} A'_{\mathfrak{p}'}$.

2. Show that any local field is the completion of a global field at an absolute value.