# Exercise sheet 21

## Structure of Local Fields, Unramified and Tame Extensions

1. Let $K$ be a nonarchimedean local field of characteristic $p > 0$ with valuation ring $\mathcal{O}$ and maximal ideal $\mathfrak{m}$. Show that the subgroup $U_1 := \{x \in \mathcal{O}^\times \mid x \equiv 1 \bmod \mathfrak{m}\}$ is topologically isomorphic to a countably infinite product of copies of $\mathbb{Z}_p$.

2. Let $\mathbb{Q}_p^{\mathrm{nr}}$ be the union of all finite unramified extensions of $\mathbb{Q}_p$. Since this is an algebraic extension of $\mathbb{Q}_p$, the valuation $\mathrm{ord}_p$ on $\mathbb{Q}_p$ extends uniquely to a valuation on $\mathbb{Q}_p^{\mathrm{nr}}$. Show that this extension is not complete.

3. Let $L$ be an algebraic extension of a nonarchimedean local field $K$.

   (a) Show that there is a maximal intermediate field $M$ that is unramified over $K$.

   (b) Show that the residue field extension of $M/L$ is trivial.

4. Show that every finite extension of $\mathbb{C}((t))$ has the form $\mathbb{C}((\sqrt[n]{t}\,))$ for some $n \geqslant 1$.