Exercise sheet 22

Tame and Wild Extensions, Ramification Filtration

1. Let K be a non-archimedean local field of characteristic $p > 0$. Show that for any integer $n \geqslant 0$, up to isomorphism there exists a unique totally inseparable extension of K of degree p^n.

2. Let K be a non-archimedean local field. Show that the maximal tame abelian extension K^{atr} of K is finite over the maximal unramified extension K^{nr} of K.

3. Let K be a non-archimedean local field of characteristic $p > 0$. Show that for every integer $s \geqslant 0$ that is not divisible by p there exists a cyclic extensions L/K with Galois group $\Gamma \cong \mathbb{F}_p$, for which $\Gamma_s = \Gamma$ and $\Gamma_{s+1} = 0$.

   (*Hint:* Study polynomials of the form $X^p - X - a$ with $v(a) < 0$.)

4. In the situation of the preceding exercise, what happens with polynomials of the form $X^p - X - a$ with $v(a) \geqslant 0$?

5. Show that a local field of characteristic zero possesses only finitely many extensions of any fixed degree, up to isomorphism.