D-MATH | Number Theory II | FS 2024
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Exercise sheet 23

Ramification Filtration

1. Let L/K be a finite Galois extension of nonarchimedean local fields with Galois group Γ.
   (a) Compute $\eta_{L/K}(s)$ for all $s \geqslant -1$ with $\Gamma_1 \subset \Gamma_s$.
   (b) Compute the upper numbering filtration of Γ when L/K is tame.
   (c) Compute the upper numbering filtration of Γ when $[L/K]$ is prime.
2. Determine the lower and upper numbering filtrations on $\mathrm{Gal}(K/\mathbb{Q}_2)$ for the following fields K:
   (a) The splitting field of the polynomial $x^2 - 2$.
   (b) The splitting field of the polynomial $x^4 - 2$.
3. Determine the lower and upper numbering filtrations on the local galois group
$$\mathrm{Gal}(\mathbb{Q}_p(\mu_{p^m})/\mathbb{Q}_p) \;\cong\; (\mathbb{Z}/p^m\mathbb{Z})^\times.$$
4. Let G be a finite group of order n, and let R be a unitary commutative ring such that n is invertible in R. Show that for any $R[G]$-module M the natural map $M^G \to M_G$ is an isomorphism.