Presence Sheet 9

Exercise 1. Consider the algebraic variety

$$X = \{M \in \mathrm{Mat}(2 \times 2, K) : \det M = 0\}.$$

a) Show that X is irreducible of dimension 3.

b) Show that the zero matrix $M = 0$ is the only singular point of X.

c) Let $\widetilde{X} = \mathrm{Bl}_0 X$ be the blow-up of X at the origin.

- *i*) Show that $\widetilde{X}$ is smooth.
- *ii*) Show that the exceptional locus is isomorphic to $\mathbb{P}^1 \times \mathbb{P}^1$.

Note: The variety X is called the *cone over the quadric surface*, and the blow-up $\widetilde{X} \to X$ is the *resolution of the conical singularity*.

Exercise 2. Recall from Presence Sheet 4 the notion of a linear algebraic group $G = (G, m, i, e)$. Show that any linear algebraic group is smooth.
Hint: Combine [Gathmann, Remark 10.20] with the strategies and results applied on Presence Sheet 4.