

## Exercise Sheet 3

1. Let  $X$  be a locally compact Hausdorff space. Show that the canonical map

$$X \rightarrow \widehat{C_0(X)}$$

is a homeomorphism where  $\widehat{C_0(X)}$  is endowed with the Gelfand topology.

2. Find an example of a commutative Banach algebra  $A$  for which the Gelfand transform  $A \rightarrow C_0(\widehat{A})$  is not surjective.
3. Consider the Banach algebra  $A := \ell^1(\mathbb{Z})$  with convolution product and

$$B := \{f \in A : f(n) = 0 \forall n < 0\}.$$

Show that  $B$  is a unital subalgebra of  $A$ . Moreover, prove  $\text{Sp}_A(\delta_1) \subsetneq \text{Sp}_B(\delta_1)$ .