# 11. Differential forms, part one

This sheet contains the first exercises involving differential forms. You can solve the exercises using material covered before chapter 9.3 in the lecture notes.

### 11.1. Characterization of orientability.

Let $M$ be a smooth $m$-dimensional manifold. Show that $M$ is orientable if and only if there exists a nowhere vanishing $m$-form on $M$.

### 11.2. Properties of the wedge product.

Prove the following two statements (see Remark 9.3 in the lecture notes): given $\omega \in \Lambda^k V^*$, $\eta \in \Lambda^l V^*$,

1. $(\omega \wedge \eta) \wedge \rho = \omega \wedge (\eta \wedge \rho)$
2. $\omega \wedge \eta = (-1)^{kl} \eta \wedge \omega$.

### 11.3. Example of pull-back.

Let $\omega \in \Omega^1(\mathbb{R}^2 \setminus \{0,0\})$ be the 1-form given by

$$\omega = -\frac{y}{x^2+y^2} dx + \frac{x}{x^2+y^2} dy,$$

and let $f$ be the map $f(r,\theta) = (r\cos(\theta), r\sin(\theta))$ defined on $\mathbb{R}^2 \setminus (-\infty, 0]$.
Compute $f^*\omega$.

### 11.4. Invariant $n$-forms on Lie groups.

Prove that the space of left (or right) invariant $n$-forms on a Lie group $(G, g)$ of dimension $n$ is a one dimensional vector space.