13. Cohomology

13.1. Cohomology of punctured plane.

Compute $H^1(\mathbb{R} \setminus \{(x_1, y_1) \cup (x_2, y_2)\})$ and provide an explicit description of its generators.

13.2. Compactly supported cohomology.

Prove Lemma 9.30 in the lecture notes.

13.3. Non-compact manifolds.

Prove that for any non-compact orientable manifold M of dimension m, $H_{dR}^m(M) = 0$

13.4. Homeomorphic manifolds.

Assuming that two smooth manifolds M and N are homeomorphic, show that $H_c^k(M) \cong H_c^k(N)$.