Talk 10: The method of moments

Speakers. Nicolas Hotton & Jonathan Wächter
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Reference material

[Dur19, Bilon, RP23]

Goal

The goal is to see how one can show convergence in distribution for real-valued random variables using convergence of moments. This is based on the fundamental probabilistic notion of "tightness".

Content

- Helly's selection theorem (Theorem 3.2.12 in [Dur19]) and tightness (Theorem 3.2.13 in [Dur19])

- Application of tightness for showing convergence in distribution (Corollary p337 in Section 25 in [Bilon])

- Skorokhod's representation theorem for real-valued random variables (Theorem 25.6 in [Bilon], but follow the proof given in [RP23] which I find more readable, it is the beginning of the proof of Proposition 1.50 there, see Remark 1.53) and a consequence (Theorem 25.12 in [Bilon]; it is not needed to prove Theorem 16.14 on uniform integrability)

- The moment problem (Theorem 30.1 in [Bilon])

- The method of moments (Theorem 30.2 in [Bilon])

References

[Bilon] Patrick Billingsley. *Probability and measure*. John Wiley & Sons, https://www.colorado.edu/amath/sites/default/files/attached-files/billingsley.pdf, 2017, Third Edition.

[Dur19] Rick Durrett. *Probability: theory and examples*, volume 49. Cambridge university press, https://services.math.duke.edu/~rtd/PTE/PTE5_011119.pdf, 2019.

[RP23] Sheldon M Ross and Erol A Peköz. *A second course in probability*. Cambridge University Press, https://people.bu.edu/pekoz/A_Second_Course_in_Probability-Ross-Pekoz.pdf, 2023.