ETH Zürich, SS 2024
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Introduction to Mathematical Finance

Exercise sheet 2

Please submit your solutions online until Wednesday 22:00, 06/03/2024.

Exercise 2.1 Denote by $\mathcal{P}$ the collection of all EMMs and denote by $|\mathcal{P}|$ the cardinality of $\mathcal{P}$. Prove or disprove:

$$\text{Completeness} \iff |\mathcal{P}| \leq 1.$$

Exercise 2.2 Let D^0 be a numéraire, and define

$$\Psi^Q(c) = c_0 + E_Q\left[\frac{c_T}{D^0}\right] \quad \text{for } c \in \mathcal{C},$$
$$Q^\Psi[A] = \Psi(0, D^0 \mathbb{1}_A) \quad \text{for } A \in \mathcal{F}.$$

Here Q is a probability measure on $\mathcal{F}$, and Ψ is a linear functional in $\mathcal{C}$. Denote by J the mapping : $Q \mapsto J(Q) := \Psi^Q$. If Ψ is a consistent price system, check that $J(\hat{Q}) = \hat{\Psi}$.

Exercise 2.3 Consider the one-step *binomial market* defined by

$$\pi = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad \text{and} \quad \mathcal{D} = \begin{pmatrix} 1+r & 1+u \\ 1+r & 1+d \end{pmatrix}$$

for some $r > -1$ and $u > d$.

(a) Show that this market is free of arbitrage if and only if $u > r > d$,

(b) Construct an arbitrage opportunity for a market where $u = r > d$.