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Introduction to Mathematical Finance

Exercise sheet 4

Please submit your solutions online until Wednesday 22:00, 20/03/2024.

Exercise 4.1

Consider a model (with a numéraire) with $d = 1$ traded risky asset X, with $X_0 = 1$ and

$$\Delta X_t = \eta_t, \qquad t = 1, 2, 3,$$

where the η_t are i.i.d. $\eta_1 \sim \mathcal{N}(0, 1)$-distributed.

(a) Suppose that a trader decides at time $t = 0$ to buy 2 shares, to sell 3 shares at $t = 1$ and then to buy 1 share at time $t = 2$. Denote by G_t his cumulative gain from the corresponding self-financing trading strategy. Find the distribution of G_3.

(b) Suppose that $\mathcal{F}_t = \sigma(S_1, \ldots, S_t)$ for $t = 1, 2, 3$. Show that there is no arbitrage in this model.

Exercise 4.2

Consider a market with trading dates $t = 0, \ldots, T$, with N traded assets on the probability space $(\Omega, \mathcal{F}, P)$ and a filtration $\mathbb{F} = (\mathcal{F}_t)_{t=0,\ldots,T}$, i.e., a general multiperiod market.

For any strategy ψ, we define the process $\widetilde{C}(\psi) = (\widetilde{C}_t)_{t=0,\ldots,T}(\psi)$ by

$$\widetilde{C}_t(\psi) := \widetilde{V}_t(\psi) - \widetilde{G}_t(\psi).$$

The process $\widetilde{C}$ is called the *cost process* for ψ.

(a) Show that

$$\Delta\widetilde{C}_{t+1}(\psi) = \Delta\psi_{t+1} \cdot S_t,$$

for $t = 1, \ldots, T - 1$.

(b) Show that ψ is self-financing if and only if

$$\widetilde{C}_t(\psi) = \widetilde{C}_0(\psi)$$

for $t = 1, \ldots, T$.

Hint: Be careful with definitions at the first time point.

Exercise 4.3 Consider the standard model for a financial market in finite discrete time with a numéraire S^0.

(a) Show that a strategy ψ is self-financing for S if and only if it is self-financing for S/S^0.

(b) Show that S satisfies NA if and only if S/S^0 satisfies NA.