IMF - SAMPLE OF EXAM QUESTIONS

PROF. BEATRICE ACCIAIO

Important information

- All material covered during lectures and exercise classes is examinable.
- For each student, the **first** question at the exam will be *either* one of the exercises done during the exercise classes, *or* one question from the list below.
- This concerns only the first question. The rest of the exam will consist of questions regarding the whole material, as well as unseen exercises.

Sample of questions

CHAPTER 1

(1) Let Ω be finite. Introduce the concept of a preference order for agents, and its axioms. Introduce the concepts of budget set and equilibrium.
(2) Let Ω be finite. Introduce the concepts of feasible allocation, Pareto efficient allocation, and attainability of a consumption process, and their meaning.
(3) Let Ω be finite. Explain the main steps of the proof of the following result: in a complete market, any equilibrium allocation is Pareto efficient.
(4) Let Ω be finite. Explain the main steps of the proof of the following result: in equilibrium, there are no arbitrage opportunities.
(5) Let Ω be finite. State the relation between NA and existence of EMM and explain the main steps of the proof.
(6) Let Ω be finite. Give the definition of complete market. Give its characterization in terms of EMM and explain the main steps of the proof.
(7) Let Ω be finite. Give the definition of consistent price system and its relation to EMM, explaining the main steps of the proof.
(8) Let Ω be finite. Give the definition of attainable payoff. List equivalent characterizations and explain the main steps of the proof.
(9) Let Ω be finite. Give the definitions of seller and buyer prices for a claim. State their relation with attainability of the claim and explain the main steps of the proof.
(10) Let Ω be finite. State the pricing-hedging duality result and explain the main steps of the proof.
(11) Let Ω be finite. State the relation between the price of a contingent claim and the seller and buyer prices, distinguishing between replicable and non-replicable claims. Explain the main steps of the proof.

CHAPTER 2

(1) Give the mathematical formulation of a multiperiod financial market, including the concepts of self-financing trading strategy and wealth process.
(2) Give the definition of an arbitrage opportunity, explaining all intervening concepts (e.g. admissibility).
(3) Give the definitions of EMM and ELMM and their financial meaning.
(4) Show that $\mathcal{P}_{loc} \neq \emptyset$ implies NA.
(5) State the DMW corollary and the theorem from which it follows, explaining the main steps of the proof of the latter.
(6) Give the definition of density process of a probability measure $\mathbb{Q}$ w.r.t. another probability measure $\mathbb{P}$, explain some of its main properties, and how $\mathbb{P}$-martingales relate to $\mathbb{Q}$-martingales.
(7) Give the definition of attainable payoff. List equivalent characterizations and explain the main steps of the proof.
(8) Give the definition of complete market. Give its characterization in terms of ELMM and explain the main steps of the proof.
(9) Describe the binomial model and its main features.
(10) State the optional decomposition theorem and explain the main steps of the proof.
(11) State the hedging duality theorem and explain the main steps of the proof.

CHAPTER 3

(1) Explain the utility maximization problem, with and without consumption, providing the mathematical formulations.
(2) Let $\Gamma_k(\theta', c') := \mathbb{E}\left[\sum_{j=0}^{T} U_c(c'_j) + U_W(W_T^{V_0,\theta',c'} - c'_T) | F_k\right]$ and $J_k(\theta, c) := \text{ess sup}\{\Gamma_k(\theta', c') : (\theta', c') \in \mathcal{A}_k(\theta, c)\}$. State conditions under which $J(\theta, c)$ satisfies the super-martingale property, and give a sketch of the proof.
What is a further condition to ensure that $J(\theta, c)$ is a super-martingale?
(3) State the martingale optimality principle and explain the main steps of the proof.
(4) Explain the recursive algorithm for constructing a candidate for the optimizer in the utility maximization problem (dynamic programming).
(5) Give an illustration of MOP and DP (example with independent returns and power utility).

CHAPTER 4

(1) Introduce the set $\mathcal{C}(x) = \mathcal{C}^x_{adm}(x)$ of non-negative final positions one can superreplicate from initial wealth x following some 0-admissible self-financing strategy. Explain how the optimal value $u(x)$ of the utility maximization problem can be written in terms of $\mathcal{C}(x)$.
(2) Introduce the set $\mathcal{Z}(z)$ and formulate the dual problem $j(z)$ in terms of $\mathcal{Z}(z)$. Introduce the set $\mathcal{D}(z)$ explaining why the dual problem can be expressed in terms of $\mathcal{D}(z)$.
(3) Explain the main steps of the proof of the fact that $\mathcal{D}(z)$ is convex and closed in L^0 (in particular, mention the importance and use of Komlos' lemma).
(4) State the theorem about uniqueness of the solution to the dual problem $j(z)$ and explain the main steps of the proof.

(5) State some properties of the function j (value function of the dual problem) and explain the main steps of the proof.
(6) Give the 4 steps of the recipe for finding a primal solution, by first solving the dual problem.
(7) Give the definition of asymptotic elasticity and examples of utility functions for which it works, resp. does not work. Give the definition of Inada conditions. What can we deduce on $j(z)$ under those conditions?
(8) State the duality relation result between $j(z)$ and $u(x)$ and provide an idea of the proof.
(9) State smoothness result for the function j (value function of the dual problem) and provide an idea of the proof.
(10) Recall conditions under which it holds that $f \in \mathcal{C}(x)$ IFF $\sup_{h\in\mathcal{D}(1)} \mathbb{E}[fh] \leq x$ and explain the main steps of the proof.
(11) State the theorem about uniqueness of the solution to the (primal) utility maximization problem and its relation to the unique solution of the dual problem. Provide the proof.
(12) Give the idea of a direct solution to the (primal) utility maximization problem and explain its limitations.