# Mathematics of New Technologies in Finance

## Exercise sheet 5

**Exercise 5.1 (Self financing portfolio)** Recall the definition of self financing trading strategy $\xi$ and its associated discounted value process $V = (V_t)_{t=0,\dots,T}$ is given by

$$V_0 := \xi_1 \cdot \bar{X}_0 \quad \text{and} \quad V_t := \xi_t \cdot \bar{X}_t \quad \text{for } t = 1, \dots, T.$$

The gains process associated with $\xi$ is defined as

$$G_0 := 0 \quad \text{and} \quad G_t := \sum^{t} \xi_k \cdot (X_k - X_{k-1}) \quad \text{for } t = 1, \dots, T$$

(a) Prove $\xi_t \cdot \bar{X}_t = \xi_{t+1} \cdot \bar{X}_t$ for $t = 1, ..., T-1$.

(b) Prove $V_t = V_0 + G_t = \xi_1 \cdot \bar{X}_0 + \sum_{k=1}^{t} \xi_k \cdot (X_k - X_{k-1})$ for all $t$.

**Exercise 5.2 (Options)** Code option pricing and simulation for European options and Digital options, see exercise notebook.

**Exercise 5.3 (Heston Model Simulation)** Code Heston Model Simulation, see exercise notebook.

## References

[1] Hans Buehler, Lukas Gonon, Josef Teichmann, and Ben Wood. Deep hedging. *Quantitative Finance*, 19(8):1271–1291, 2019.

[2] Hans Föllmer and Alexander Schied. *Stochastic finance*. de Gruyter, 2016.