Mathematics for New Technologies in Finance

Exercise sheet 6

Exercise 6.1 (Bayesian optimization)

(a) Recall the definition of prior, likelihood, posterior, and evidence distributions in bayesian statistics.

(b) Consider linear model on $\mathbb{R} : Y \sim \theta X + Z, \theta \sim \mathcal{N}(0,1), Z \sim \mathcal{N}(0,1)$ and θ independent with X. Compute $p_\theta(y \mid x)$ and $p(\theta \mid x, y)$. Prove that maximizing the posterior $p(\theta \mid x, y)$ is exactly doing Ridge regression (fix λ here).

(c) Consider Lasso regression, what is the prior under Bayesian perspective? Please calculate the posterior under this prior.

(d) Would you expect a sparser weight or denser weight using Lasso regression instead of Ridge regression.

Exercise 6.2 (Implied volatility) The Black-Scholes formula provides a relationship between the price of a European Call option $C(K,T)$ and volatility $\sigma(K,T)$ for fixed price of underlying S_0, strike K, and maturity T. It is an important transformation in Finance to calculate from $C(K,T)$ the *implied volatility* $\sigma(K,T)$. Proceed in the following steps:

- Define a Gamma prior on implied volatility.
- Define a likelihood, which predicts the price given an implied volatility.
- Construct a posterior via Bayes formula and sample from it via Langevin dynamics. Interpret the resulting algorithm from the perspective of stochastic gradient descent.

References

[1] Trevor Hastie, Robert Tibshirani, Jerome H Friedman, and Jerome H Friedman. *The elements of statistical learning: data mining, inference, and prediction*, volume 2. Springer, 2009.