Mathematics of New Technologies in Finance

Solution sheet 5

Exercise 5.1 (Self financing portfolio) Recall the definition of self financing trading strategy ξ and its associated discounted value process $V = (V_t)_{t=0,\dots,T}$ is given by

$$V_0 := \xi_1 \cdot X_0 \quad \text{and} \quad V_t := \xi_t \cdot X_t \quad \text{for } t = 1, \dots, T.$$

The gains process associated with ξ is defined as

$$G_0 := 0 \quad \text{and} \quad G_t := \sum^{t} \xi_k \cdot (X_k - X_{k-1}) \quad \text{for } t = 1, \dots, T$$

(a) Prove $\xi_t \cdot X_t = \xi_{t+1} \cdot X_t$ for $t = 1, ..., T-1$.

(b) Prove $V_t = V_0 + G_t = \xi_1 \cdot X_0 + \sum_{k=1}^{t} \xi_k \cdot (X_k - X_{k-1})$ for all t.

Solution 5.1

(a) By definition we have

$$\xi_t \cdot S_t = \xi_{t+1} \cdot S_t \text{ for } t = 1, ..., T-1,$$

S_t is the price of the asset at time t. By dividing both sides by S_t^0, we can prove $\xi_t \cdot X_t = \xi_{t+1} \cdot X_t$ for $t = 1, ..., T-1$.

(b) Since (a) holds, we have

$$\xi_{t+1} \cdot X_{t+1} - \xi_t \cdot X_t = \xi_{t+1} \cdot X_{t+1} - \xi_{t+1} \cdot X_t = \xi_{t+1} \cdot (X_{t+1} - X_t)$$

for $t = 1, ..., T-1$, and it's identical to (b).

Exercise 5.2 (Options) Code option pricing and simulation for European options and Digital options, see exercise notebook.

Solution 5.2 See notebook.

Exercise 5.3 (Heston Model Simulation) Code Heston Model Simulation, see exercise notebook.

Solution 5.3 See notebook.

References

[1] Hans Buehler, Lukas Gonon, Josef Teichmann, and Ben Wood. Deep hedging. *Quantitative Finance*, 19(8):1271–1291, 2019.

[2] Hans Föllmer and Alexander Schied. *Stochastic finance*. de Gruyter, 2016.