Exercise 2 for March 7th

Exercise. Let X be a real-valued random variable. What can be said on the asymptotic behavior of $\mathbb{P}(X \geq x)$ as $x \to \infty$ in the following two cases?

(1) X satisfies $\mathbb{E}\left[e^{tX}\right] < \infty$ for every $t \in \mathbb{R}$ (Cramer condition)

(2) X satisfies $\mathbb{E}\left[e^{tX}\right] < \infty$ for t in a neighborhood around 0 ("local" Cramer condition).

Remark. The question is formulated in an open way, there is no unique answer.