Exercise 3 for March 14th

Exercise. Let $(X_i)_{i\geq 1}$ be i.i.d. random variables. Assume that X_1 is integer-valued, aperiodic, satisfies Cramer condition, with $\mathbb{E}[X_1] = 0$ and has positive variance. Set $S_n = X_1 + \cdots + X_n$. Show that

$$\frac{1}{n}\sum_{k=1}^{n}|X_k| \quad \text{under} \quad \mathbb{P}\left(\cdot \mid S_n=0\right)$$

converges in probability.