Exercise 4 for April 11

Exercise. Let $(X_i)_{i\geq 1}$ be i.i.d. random variables. Assume that X_1 is integer-valued non-constant, aperiodic, with $\mathbb{E}\left[X_1^2\right] < \infty$ and $\mathbb{E}\left[X_1\right] = 0$ Set $S_n = X_1 + \cdots + X_n$.

(1) Show that $\dfrac{\max(X_1, \ldots, X_n)}{\sqrt{n}}$ converges to 0 in probability.

(2) Show that the same result holds under the conditional probability $\mathbb{P}(\cdot \mid S_n = 0)$.

Hint. First show that $\dfrac{\max(X_1, \ldots, X_{\lfloor n/2 \rfloor})}{\sqrt{n}}$ converges to 0 in probability under the conditional probability $\mathbb{P}(\cdot \mid S_n = 0)$.