$$\boxed{\text{Exercise 6 for April 25}}$$

Exercise. This exercise is made of two independent questions.

(1) In the proof of the Theorem of Lecture 5, can one replace $\ln(m)^3$ by $\ln(m)$ (and by adding some constants when needed)?

(2) Assume that X satisfies assumption (H_Δ) (with $T < \infty$ or $T = \infty$). Let X_1, X_2 be independent with same law as X. Is it true that

$$\mathbb{P}\left(X_1 + X_2 \geq u\right) \underset{u \to \infty}{\sim} 2\mathbb{P}\left(X_1 \geq u\right)?$$