Exercise. Let $(X_i)_{i\geq 1}$ be a sequence i.i.d. random variables with $\mathbb{E}[X_1] = 0$, $\mathbb{E}[X_1^2 \mathbb{1}_{X_1\leq 0}] < \infty$ and $\mathbb{P}(X_1 \geq u) \sim c/u^{\beta}$ as $u \to \infty$ with $\beta > 2$. Set $S_n = X_1 + \dots + X_n$. Let $(x_n)_{n\geq 1}$ be a sequence such that $\liminf_{n\to\infty} x_n/n > 0$. Obtain a limit theorem for $\max(X_1, \dots, X_n)$ under $\mathbb{P}(\cdot|S_n \geq x_n)$ as $n \to \infty$.

Set
$$M_n = \max(X_1, ..., X_n)$$
. Intuition: By the one big-jump principle we should have
 $B(M_n > M \times n) \le n B(X_1 > M \times n)$ for a large. In addition $B(S_n > M \times n) \le n B(X_1 > M \times n)$
Thus we exped, for $M \le 1$, $\frac{B(M_n > M \times n)}{B(S_n > 2n)} \xrightarrow{1}{M^p}$
We show that $\frac{H_n}{\pi_n}$ converges indestribution to T with density. $\frac{P}{M^{p+1}} \int_{M \times 1} \int_{M \times 1} \int_{M} \int_{M \times 1} \int_$