$$\boxed{\text{Exercise 9 for May 23th}}$$

Exercise. Let μ be a subcritical offspring distribution (that is $\sum_{k=0}^{\infty} k\mu(k) < 1$) such that $G_\mu(x) = \sum_{k=0}^{\infty} \mu(k)x^k$ has an infinite radius of convergence. Show that there exists $b > 0$ such that $\hat{\mu}$ defined by

$$\hat{\mu}(k) = \frac{b^k \mu(k)}{G_\mu(b)}$$

is critical (that is $\sum_{k=0}^{\infty} k\hat{\mu}(k) = 1$).