

INTEGRATION PART II AND COMPLEX NUMBERS PART I

1. Write the following expressions in the form $x + iy$, with $x, y \in \mathbb{R}$.

$$\begin{array}{lll} \text{(a)} & (3 + 5i)^2 & \text{(b)} \quad (-7 + 2i)(5 - 3i) & \text{(c)} \quad |2 + i| \\ \text{(d)} & \frac{i - 1}{1 + i} & \text{(e)} \quad \frac{1 - 5i}{3i - 1} & \text{(f)} \quad \sqrt{2}e^{\frac{\pi i}{4}} \end{array}$$

2. Can you give formulas for the loci $z \in \mathbb{C}$ having the following shapes:

- (a) The upper right quadrant, excluding the axes,
- (b) a circle of radius 2 centered at $z = i$,
- (c) the perpendicular bisector to the segment between the points $z = a$ and $z = b$, where $a, b \in \mathbb{C}$ are fixed,
- (d) five points evenly spread out on the unit circle, with one of the points being $z = 1$.

3. Compute the following integrals by using the method of partial fraction decomposition.

$$\text{(a)} \int_2^3 \frac{x - 1}{x(x^2 - 2)} dx, \quad \text{(b)} \int_{-1}^1 \frac{x^2}{(x + 2)(x + 3)^2} dx,$$

4. Compute the following integrals by choosing the right method. Check your solutions by differentiating the results.

$$\begin{array}{ll} \text{(a)} \int 2x \sin(x^2 + 1) dx, & \text{(b)} \int x \cos x \sin x dx, \\ \text{(c)} \int \frac{1}{x^2(x^2 - 1)} dx, & \text{(d)} \int \frac{1}{x \ln x}. \end{array}$$

5. What advantages might the complex numbers have over the reals?

6. An extended period of rain in an area causes a waterflow. After t days, the flow is $f(t)$ mm of water per day, modelled by the function

$$f(t) = 25e^{-t} + e^{-0.05t}.$$

How much water flows from the area in the first ten days?