DIFFERENTIAL CALCULUS

1. Compute the first derivative of

$$\text{(a) } x^8 e^{-x^3} - x - 100, \qquad \text{(b) } \frac{\ln(\sin^2(x))}{\cos(x)}, \qquad \text{(c) } \arctan(\sqrt{x}).$$

   (Hint for (c): Set $\tan(y) = \sqrt{x}$ and differentiate with respect to x)

2. Compute the first derivative of the two functions

   (a) $f(x) = e^{\sin(x^3+\cos(x^2))}$,

   (b) $g(x) = \cos^2\left(\frac{x^3+1}{x^2+1}\right)$.

3. For which $x \in \mathbb{R}$ does the graph of $f : \mathbb{R} \to \mathbb{R}$ with

$$y = f(x) = e^{\sin x} \cdot e^{\cos x}$$

   have horizontal tangents? These are tangents of the form $t(x) = a$ with $a \in \mathbb{R}$.

4. What is the domain of the function of the function

$$h(x) = \ln(\ln(x))?$$

   Compute the second derivative $h''(x) = (h'(x))'$. Does the function h have any inflection points (i.e. points where h'' changes sign)?

5. Use the definition of the derivative to

   (a) differentiate x^3,

   (b) prove the product rule $(fg)' = f'g + fg'$.

6. Find the derivative of x^x. (Hint: Rewrite the expression as $e^{\text{something}}$)

7. (Optional) Have a listen: https://www.youtube.com/watch?v=tSovvlCxUNs
   (You don't need to know German for this.)

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