DIFFERENTIAL CALCULUS

All questions using MATLAB are optional. We encourage you to familiarize yourself with MATLAB, as you might ahve to use it later in other courses. For installation, you can follow his tutorial https://www2.math.ethz.ch/education/bachelor/lectures/fs2013/other/num_meth_itet/Matlab.html

1. Plot the two functions $f(x) = 4x^3 + 2x^2 - 5x - 2$ and $g(x) = 2x^2 - x - 2$ for $x \in [-2, 2]$ in MATLAB, using the command `fplot`.
   For help, see https://ch.mathworks.com/help/matlab/ref/fplot.html.

   (a) Determine the three x-coordinates $x_1 < x_2 < x_3$ of the points, where the graphs of the two functions $f(x)$ and $g(x)$ intersect.

   (b) On the x-axis of your plot, let $x_{\min}$ and $x_{\max}$ correspond to the local minimum and the local maximum of f. Compute by hand $x_{\min}$ and $x_{\max}$.

2. Find the global minimum and the global maximum of the function

$$f(x) = x^4 - 4x^3 + 4x^2 - 3$$

   on the interval $[-2, 3]$. Sketch the function by hand (and if you want, using MATLAB).

3. Find the equation of the line that is perpendicular to the tangent to the curve

$$y(x) = (x^4 - 1)^3 \ln(x + 1)$$

   at the origin. Sketching by hand the curve first will help.

4. a) Suppose that a function f is continuous and differentiable in the interval $[0, 1]$. Suppose further that $f(0) = -1$ and $f'(x) \leq 2$ for all $x \in [0, 1]$. What is the largest possible value of $f(1)$? (Hint: Use the Mean Value Theorem)

   b) Let $f : [0, 1] \to [0, 1]$ be a continuous function. Prove that f has a fixed point, i.e. $f(x) = x$ for some $x \in [0, 1]$ (Hint: Intermediate Value Theorem).

5. Determine the local extrema of the function $f(x) = x(x - 1)^4$, and sketch the graph of f. Are there any global extrema?

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