INTEGRATION PART 1

1. Compute the following integrals by finding an appropriate antiderivative.

   (a) $\int_{-2}^{2} (x^3 + 8x)\,\mathrm{d}x$ (b) $\int e^{-7x}\,\mathrm{d}x$ (c) $\int \sqrt{5x}\,\mathrm{d}x$

   (d) $\int_{0}^{\infty} \frac{e^{-\sqrt{x}}}{\sqrt{x}}\,\mathrm{d}x$ (e) $\int_{2}^{8} \frac{1}{x}\,\mathrm{d}x$ (f) $\int \mathrm{d}x$

2. Compute the following integrals using integration by parts.

   (a) $\int \cos x \ln(\sin x)\,\mathrm{d}x$ (b) $\int \frac{x}{\cos^2 x}\,\mathrm{d}x$ (c) $\int x^3 e^x\,\mathrm{d}x$

   (d) $\int \ln(x^2 + 1)\,\mathrm{d}x$ (e) $\int x \ln x\,\mathrm{d}x$ (f) $\int \sin^2 x\,\mathrm{d}x.$

   Bonus: can you find $\int_0^{\pi/2} \sin^2 x dx$ directly, without using integration by parts?

3. The picture below shows the graphs of the functions

$$f(x) = 4x^3 + 2x^2 - 5x - 2 \qquad \text{and} \qquad g(x) = 2x^2 - x - 2.$$

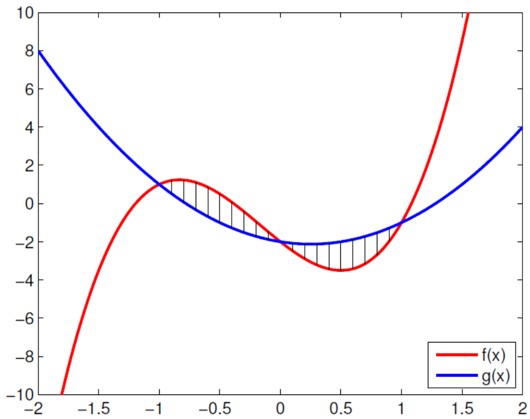

Figure 1: The graphs of the two functions f and g.

   (a) Determine the x-coordinates $x_1 < x_2 < x_3$ of the points where the graphs intersect.

   (b) Calculate the integral $\int_{x_1}^{x_3} \big(f(x) - g(x)\big)\,\mathrm{d}x.$

   (c) Calculate the area of the shaded region.

4. (Optional: Food for thought...) Let $f\colon [0,1] \to \mathbb{R}$ be a continuous function such that
$$\int_0^1 f(x)g(x)dx = 0$$
for all continuous functions $g\colon [0,1] \to \mathbb{R}$ with $g(0) = g(1) = 0$. Prove that f must be identically zero.

5. For which $x \in (0, \frac{3\pi}{2})$ is $f(x) = \int_x^{2x} \frac{\sin t}{t} dt$ a local maximum?