

## Exponential function and logarithm

Exponential function                       $\exp: x \mapsto e^x$   
 Natural logarithm                          $\ln: u \mapsto \ln u$

### Relations

$$e^x = u \iff x = \ln u \quad (u > 0)$$

$e^x = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n$ $\exp'(x) = \exp(x) = e^x$	$\ln u = \int_1^u \frac{1}{t} dt$ $\ln'(u) = \frac{1}{u}$
$e^0 = 1$ $e^1 = e$ $e^{x+y} = e^x \cdot e^y$ $e^{-x} = \frac{1}{e^x}$ $(e^x)^y = e^{xy}$	$\ln 1 = 0$ $\ln e = 1$ $\ln(uv) = \ln u + \ln v$ $\ln \frac{1}{u} = -\ln u$ $\ln(u^v) = v \ln u$

## Exponential function and logarithm of base $a$    ( $a > 0, a \neq 1$ )

$$a^x = u \iff x = \log_a u \quad (u > 0)$$

### Change of base

$a^x = e^{x \ln a} = 10^{x \log_{10} a} = b^{x \log_b a}$
$\log_a u = \frac{\ln u}{\ln a} = \frac{\log_{10} u}{\log_{10} a} = \frac{\log_b u}{\log_b a}$