ETH Zürich
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Mathematics

Trigonometric functions

For any $\alpha \in \mathbb{R}$:

$$e^{i\alpha} = \cos\alpha + i\sin\alpha$$

Periodicity

For $k \in \mathbb{Z}$ we have

$$\sin(\alpha + k\,2\pi) = \sin\alpha \qquad \cos(\alpha + k\,2\pi) = \cos\alpha$$

$$\tan(\alpha + k\,2\pi) = \tan\alpha \qquad \cot(\alpha + k\,2\pi) = \cot\alpha$$

Evaluate

$\sin(-\alpha) = -\sin\alpha$	$\cos(-\alpha) = \cos\alpha$	$\tan(-\alpha) = -\tan\alpha$
$\sin(\pi - \alpha) = \sin\alpha$	$\cos(\pi - \alpha) = -\cos\alpha$	$\tan(\pi - \alpha) = -\tan\alpha$
$\sin(\pi + \alpha) = -\sin\alpha$	$\cos(\pi + \alpha) = -\cos\alpha$	$\tan(\pi + \alpha) = \tan\alpha$

x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\sin x$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos x$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\tan x$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	
$\cot x$		$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0

Relations between functions of same angle

$$\cos^2\alpha + \sin^2\alpha = 1 \qquad \tan\alpha = \frac{\sin\alpha}{\cos\alpha} \qquad \frac{1}{\cos^2\alpha} = 1 + \tan^2\alpha$$

$$\cot\alpha = \frac{1}{\tan\alpha} \qquad \frac{1}{\sin^2\alpha} = 1 + \cot^2\alpha$$

Addition, subtraction and multiples of angles

$$\sin(\alpha+\beta) = \sin\alpha\,\cos\beta + \cos\alpha\,\sin\beta$$

$$\cos(\alpha+\beta) = \cos\alpha\,\cos\beta - \sin\alpha\,\sin\beta$$

$$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\,\tan\beta}$$

$$\sin(\alpha-\beta) = \sin\alpha\,\cos\beta - \cos\alpha\,\sin\beta$$

$$\cos(\alpha-\beta) = \cos\alpha\,\cos\beta + \sin\alpha\,\sin\beta$$

$$\tan(\alpha-\beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\,\tan\beta}$$

$$\sin 2\alpha = 2\,\sin\alpha\,\cos\alpha \qquad \cos 2\alpha = \cos^2\alpha - \sin^2\alpha = 1 - 2\,\sin^2\alpha = 2\,\cos^2\alpha - 1$$

$$\tan 2\alpha = \frac{2\,\tan\alpha}{1 - \tan^2\alpha}$$

$$\sin^2\frac{\alpha}{2} = \frac{1-\cos\alpha}{2} \qquad \cos^2\frac{\alpha}{2} = \frac{1+\cos\alpha}{2} \qquad \tan^2\frac{\alpha}{2} = \frac{1-\cos\alpha}{1+\cos\alpha}$$

$$\tan\frac{\alpha}{2} = \frac{1-\cos\alpha}{\sin\alpha} = \frac{\sin\alpha}{1+\cos\alpha}$$

Sum of trigonometric functions

$$\sin\alpha + \sin\beta = 2\,\sin\frac{\alpha+\beta}{2}\,\cos\frac{\alpha-\beta}{2}$$

$$\cos\alpha + \cos\beta = 2\,\cos\frac{\alpha+\beta}{2}\,\cos\frac{\alpha-\beta}{2}$$

$$\tan\alpha + \tan\beta = \frac{\sin(\alpha+\beta)}{\cos\alpha\cdot\cos\beta}$$

$$\sin\alpha - \sin\beta = 2\,\sin\frac{\alpha-\beta}{2}\,\cos\frac{\alpha+\beta}{2}$$

$$\cos\alpha - \cos\beta = -2\,\sin\frac{\alpha-\beta}{2}\,\sin\frac{\alpha+\beta}{2}$$

$$\tan\alpha - \tan\beta = \frac{\sin(\alpha-\beta)}{\cos\alpha\cdot\cos\beta}$$