1.1. Classification of PDEs Determine the order of the following PDEs. Determine also whether they are linear or not. If they are linear, determine if they are homogeneous or not, and if they are not linear, determine if they are quasilinear.

(a) $\Delta(\Delta u) = 5u$.

(b) $10^{20}u + \sin(u_x) = u_{xx}$.

(c) $e^{\Delta u} = u$.

(d) $\partial_x(uu_y) = \partial_y(uu_x)$.

1.2. Solutions to PDEs Check whether each of the following PDEs has a solution u that is a polynomial and, if it exists, determine a polynomial that solves the PDE.

(a) $\Delta u = x + y$.

(b) $u_{xx} = -u$, with $u(0) = 1$.

(c) $u_{xx} + u_{xy} = \sin(x)$.

(d) $u_{xyx}^2 + u_{yxy} = e^u$.

(e) $u_{xx} + u_y + u_{xy} = x^2 y$.

1.3. Solutions to ODEs Solve the following ODEs.

(a) $x'(t) + \lambda x(t) = 0$, with $x(0) = x_0$.

(b) $x'(t) + \lambda x(t) = 1$, with $x(0) = x_0$.

(c) $x'(t) + x(t) = t$, with $x(0) = 1$.

(d) $x'(t) + x(t) = e^t$, with $x(0) = 1$.

(e) $x''(t) + \lambda^2 x(t) = 0$, find a general solution.

1.4. Nonexistence of solutions Show that there is not a smooth function $u : \mathbb{R}^2 \to \mathbb{R}$ such that

$$\begin{cases} u_x = xy\,, \\ u_y = x^2\,. \end{cases}$$

1.5. Multiple Choice Determine the *unique* correct answer to each point.

(a) Consider the PDE $\Delta u + \nabla u \cdot \nabla u = 0$, where with the "dot" we denote the usual scalar product in $\mathbb{R}^n$. Then, the function $v := e^u$ solves a PDE that is

☐ Linear

☐ Quasi linear

☐ Not explicit

(Hint: start by computing v_{x_i} and $v_{x_i x_i}$)

(b) For a smooth vector field $F = (F^1, F^2, \ldots, F^n) : \mathbb{R}^n \to \mathbb{R}^n$, we denote with $\mathrm{div}(F)$ the divergence of F, given by $\mathrm{div}(F) := \sum_{i=1}^n F^i_{x_i}$. The following PDE

$$\mathrm{div}(\nabla(u^2)) = u,$$

is

☐ Fully non linear

☐ Linear

☐ Quasi linear