2.1. Method of characteristics Solve the following equations using the method of characteristics.

(a) $u_x + u_y = 1$, with $u(x, 0) = f(x)$.

(b) $xu_x + (x + y)u_y = 1$, with $u(1, y) = y^2$.

(c) $u_x - 2xyu_y = 0$, with $u(0, y) = y$.

(d) $yu_x - xu_y = 0$, with $u(x, 0) = g(x^2)$ for all $x > 0$.

2.2. Find a solution Consider the PDE

$$xu_x + yu_y = -2u.$$

Find a solution to the previous PDE such that $u \equiv 1$ on the unit circle.

2.3. Multiple choice Cross the correct answer(s).

(a) The expression $f(u_{xxx}) = u_z + 5$ describes a quasilinear PDE of order 3 if and only if

- ☐ f is linear
- ☐ f is invertible
- ☐ f is constant

(b) Consider the PDE $yu_x - x^2u_y = 0$ coupled with the boundary condition $u(x, y) = 2$ on $\{(x, y) : x^3 + 1 = y\}$. Then, the initial curve $\Gamma(s) = \{x_o(s), y_0(s), \tilde{u}_0(s)\}$ needed to start applying the Method of Characteristic is given by

- ☐ $x_0(s) = s^3 + 1$, $y_0(s) = s$ and $\tilde{u}_0(s) = 2$, $s \in \mathbb{R}$
- ☐ $x_0(s) = s$, $y_0(s) = s^3 + 1$ and $\tilde{u}_0(s) = 2$, $s \in \mathbb{R}$
- ☐ $x_0(s) = s^{1/3}$, $y_0(s) = s + 1$ and $\tilde{u}_0(s) = 2$, $s \in \mathbb{R}$