3.1. Characteristic method and initial conditions Consider the transport equation

$$xu_y - yu_x = 0.$$

For each of the following initial conditions, solve the problem in $y \geq 0$ whenever it is possible. If it is not, explain why.

(a) $u(x,0) = x^2$.

(b) $u(x,0) = x$.

(c) $u(x,0) = x$ for $x > 0$.

3.2. Characteristic method and transversality condition Consider the transport equation

$$yu_x + uu_y = x.$$

(a) Solve the problem with initial condition $u(s,s) = -2s$, for $s \in \mathbb{R}$. For what domain of s does the transversality condition hold?

(b) Check the transversality condition with the initial value $u(s,s) = s$. What is occurring in this case?

(c) Define

$$w_1 := x + y + u, \quad w_2 := x^2 + y^2 + u^2, \quad w_3 = xy + xu + yu.$$

Show that $w_1(w_2 - w_3)$ is constant along the characteristic curves.