Exercises with a $\star$ are eligible for bonus points. Exactly one answer to each MC question is correct.

4.1. MC Questions

(a) What's the value of $\int_0^{2\pi} e^{e^{it}+it}\,dt$?

A) 2π

B) 0

C) -2π

D) 4π

(b) Given $\gamma = \{z \in \mathbb{C} : |z| = 3\}$, what's the value of $\int_\gamma \frac{z^2+2z}{z^2-1}\,dz$?

A) $6\pi i$

B) -6π

C) $4\pi i$

D) $\frac{3}{2}\pi i$

4.2. An analytic identity Let γ be the counter-clockwise oriented circle of radius $r > 0$ and center $z_0 \in \mathbb{C}$, and let f be a function which is analytic in all of $\mathbb{C}$. Show that

$$\int_\gamma f(\bar{z})\,dz = 2\pi i r^2 f'(\bar{z}_0).$$

4.3. Let $r > 0$. Show the estimate

$$\left|\int_\gamma e^{iz^2} dz\right| \leq \frac{\pi(1-e^{-r^2})}{4r},$$

where γ is the curve with $\gamma(t) = re^{it}$, for $0 \leq t \leq \pi/4$

4.4. $\star$ **Converse of Theorem 3.2, Chapter 1** Suppose that a function f is continuous on a plane domain Ω in $\mathbb{C}$ and that $\int_\gamma f(z)dz = 0$ for every closed piecewise smooth path on Ω. Show that f has a primitive in Ω.

4.5. $\star$ **Application of Liouville's theorem** Let f and g be entire functions such that, for all $z \in \mathbb{C}$, $\Re(f(z)) \leq k\Re(g(z))$ for some real constant k, independent of z. Prove that there are constants a, b such that $f(z) = ag(z) + b$