Exercise Sheet 10

1. For each $n \geq 1$, define $\mu_n \in M^1([0,1])$ by $\mu_n = \frac{1}{n} \sum_{k=1}^{n} \delta_{k/n}$. What is the weak*-limit of $(\mu_n)_{n \geq 1}$?

2. Let Γ be a group with a normal subgroup $N \triangleleft \Gamma$ such that both N and Γ/N satisfy the conclusions of the Markoff-Kakutani fixed point theorem. Show that Γ does as well.

3. A countable group G has property (F) if there exists a sequence $F_n \subset G$, $n \geq 1$ of finite subsets with

$$\lim_{n \to \infty} \frac{|gF_n \triangle F_n\|}{|F_n|} = 0\,, \qquad \forall g \in G\,.$$

   Show that if G has property (F), it satisfies the conclusions of the Markoff-Kakutani theorem.