Exercise Sheet 11

1. Let $\alpha \notin \mathbb{Q}/\mathbb{Z}$ and let $T : \mathbb{R}/\mathbb{Z} \to \mathbb{R}/\mathbb{Z}$ send $x \mapsto x + \alpha$. Show that the uniform Lebesgue measure on $\mathbb{R}/\mathbb{Z}$ is the unique T-invariant probability measure on $\mathbb{R}/\mathbb{Z}$.

2. Let $H < G$ be groups and let G be ameanable. Show that H is also ameanable.

3. Show that $c_0(\mathbb{N})$ with the sup norm is not the dual of any normed space.

4. Let $\mathcal{M} \subset (\ell^\infty(\mathbb{N}))^*$ be the set of means, i.e. the set of elements $v \in (\ell^\infty(\mathbb{N}))^*$ such that $\forall f = (f_i)_{i=1}^\infty \in \ell^\infty$ with $f_i \geq 0$, $v(f) \geq 0$ and $v((1, 1, 1, ...)) = 1$. Characterize the extremal points of $\mathcal{M}$. Show that there are extreme points m so that $m(f) = 0$ for any function f with finite support.