Exercise Sheet 13

1. Show that if $f, g \in L^1(\mathbb{R}^n)$ then $\widehat{f * g} = \hat{f}\hat{g}$.

2. Show that if $f \in L^1(\mathbb{R}^n)$ and $f * f = f$ then $f \equiv 0$.

3. Show that if $r > 0$ and $f(x) = |x|^r$ for $x \in \mathbb{R}$ then the weak derivative of f is given by $(d/dx)_w f = r\operatorname{sgn}(x)|x|^{r-1}$, where $\operatorname{sgn}(x)$ is the sign of x.

4. Show that if $f, g \in W^{k,2}(\Omega)$ then the inner product given by

$$\langle f, g \rangle := \sum_{|\alpha| \leq k} \langle D_w^\alpha f, D_w^\alpha g \rangle_{L^2}$$

   is well-defined and the corresponding norm is equivalent to $\| \cdot \|_{k,2}$.