Exercise Sheet 6

1. Let E be a Banach space and $B^* \subset E^*$ a subset such that $\forall v \in E$,

$$\{f(u) : f \in B^*\} \subset \mathbb{K}$$

   is bounded. Show that B^* is bounded.

2. Let E, F be Banach spaces and $f : E \times F \to \mathbb{K}$ a bilinear form such that

   (a) $\forall v \in E$, $(w \in F) \mapsto f(v, w)$ is continuous.

   (b) $\forall w \in F$, $(v \in E) \mapsto f(v, w)$ is continuous.

   Show that $\exists C > 0$ such that

$$|f(v, w)| \leq C\|v\|\|w\| .$$

   Hint: Use Exercise 1.

3. Show the implication (3) $\implies$ (2) in the open mapping theorem. (Theorem 4.19)