Exercise Sheet 8

1. Let V be a topological vector space such that $\forall v \neq 0$, $\exists F \in V^*$ with $F(v) \neq 0$. Show that V is Hausdorff.

2. Let V be a finite dimensional normes space. Show that the norm topology and the weak topology on V coincide.

3. Let X be a space and $\mathcal{F} = (Y_i, \varphi_i)_{i \in \mathcal{I}}$ be a family of pairs of topological spaces Y_i with a map $\varphi_i : X \to Y_i$. Equip X with the initial topology with respect to $\mathcal{F}$. Show that a sequence $(x_n)_n \subset X$ converges to $x \in X$ if and only if $\forall i \in \mathcal{I}$, $\varphi_i(x_n)$ converges to $\varphi_i(x)$.

4. Show that on $L^2_{loc}(\mathbb{R})$ (taken with respect to the Lebesgue maesure) there is no norm inducing the topology defined in Example 5.10.