

Solution to Exercise sheet 1 Q3:

WLOG say $p_1 < p_2$. Let

$$f_n(i) := \begin{cases} n^{-1/p_2}, & \text{if } 0 < i \leq n \\ 0, & \text{otherwise} \end{cases}$$

Now check that $f_n \in c_{00}$, $\|f_n\|_{p_2} = 1$ but $\|f_n\|_{p_1} = n^{1/p_1 - 1/p_2}$. But the latter goes to $+\infty$ as $n \rightarrow \infty$. Thus the norms are not equivalent.