Solution to Exercise sheet 8 Q1:

Let $x, y \in V$. Consider $v = x - (x+y)/2$ and let $F \in V^*$ be such that $F(v) \neq 0$. If $\mathbb{K} = \mathbb{R}$, then WLOG we assume $F(v) > 0$. (The case when $\mathbb{K} = \mathbb{C}$ can be done similarly.) Now, the set $X = \{a \in V : F(a - (x+y)/2) > 0\}$ is open and x belongs to it. Now as

$$y - (x + y)/2 = (y - x)/2 = -x + (x + y)/2 = -v \,,$$

we have y belongs to the open set $Y = \{a \in V : F(a - (x + y)/2) < 0\}$. As $X \cap Y = \emptyset$, we have that V is Hausdorff.