Exercise Sheet 13

1. Pullback

Let M, N be smooth manifolds and $F\colon N \to M$ be a smooth map. For $\omega \in \Omega^s(M)$ and $\theta \in \Omega^t(M)$ prove that

a) $F^*(\omega \wedge \theta) = F^*\omega \wedge F^*\theta$,

b) $F^*(d\omega) = d(F^*\omega)$.

2. Volume Forms

Let M be an m-dimensional manifold. A *volume form* ω on M is a nowhere vanishing m-form, that is, $\omega_p \neq 0$ $(\in \Lambda_m(TM_p^*))$ for all $p \in M$.
Prove that there is a volume form on M if and only if M is orientable.

3.

Let M and N be two compact, oriented smooth manifolds of dimension m. Show that if $F, G\colon M \to N$ are smoothly homotopic maps and $\omega \in \Omega^m(N)$ is an m-form on N, then

$$\int_M F^*\omega = \int_M G^*\omega.$$