Numerical Methods for Elliptic and Parabolic Partial Differential Equations

Exercise Sheet 1

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Exercise 1. *Show that the more general Ansatz*

$$-\sum_{i,j=1}^{d} \tilde{\mathbf{A}}_{i,j}(\mathbf{x})\partial_j\partial_i u(\mathbf{x}) + \sum_{i=1}^{d} \tilde{\mathbf{b}}_i(\mathbf{x})\partial_i u(\mathbf{x}) + \tilde{c}(\mathbf{x})u(\mathbf{x}) = g(\mathbf{x})$$

for twice differentiable functions u can always be transformed to the form (1.1) of the lecture notes.

Exercise 2. *Let $\Omega \subset \mathbb{R}^2$ a two-dimensional domain. Determine (guess) three linear independent solutions to the equation*

$$-\Delta u = 4 \quad \text{in } \Omega.$$

Exercise 3. *Let $\Omega =]-1, 1[$ and $\varphi(x) = |x|$. Then φ is differentiable (in the sense of Definition 2.1 of the lecture notes) and its weak derivative reads*

$$\psi(x) = \begin{cases} -1 & -1 < x < 0, \\ 1 & 0 < x < 1. \end{cases}$$

Show that $\psi(x)$ is not weakly differentiable on $]-1, 1[$.