Numerical Methods for Elliptic and Parabolic Partial Differential Equations

Exercise Sheet 8

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Exercise 1. *A sparse matrix is defined as a matrix with a majority of its elements being zero. Consider the matrix:*

$$\mathbf{B} = \begin{bmatrix} 4 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 6 & 0 \\ 0 & 0 & 0 & 7 \end{bmatrix}.$$

(a) Find an efficient way to store a sparse matrix, where most entries are zero. How much memory does your method require, as the size of the matrix tends to infinity? Assume that the amount of non-zero entries per row remains constant.

(b) How would you compute the map $x \mapsto \mathbf{B}x$ *with a matrix that has been stored in your format?*

(c) Write $\mathbf{B}$ *in your proposed form.*

Hint: *If you are interested, you can read some details on the compressed sparse row (CSR) format.*

Exercise 2. *On the website, you find python code,* fem_tools.py *and* grid_tools .py, *(which you can use exactly like the one developed in the lecture). The only difference is in the numbering of the nodes of the mesh. Download it and use it for this exercise.*

(a) Use the finite element code to generate the stiffness matrix $\mathbf{A}$*. Visualize the sparsity pattern (see* matplotlib.pyplot.spy*). What is the relation of the mesh and the sparsity pattern?*

(b) We intend to solve the system $\mathbf{A}x = b$*. Use the Cholesky decomposition on* $\mathbf{A}$*. Take a look on the sparsity patterns of* $\mathbf{L}$*. What do you observe?*