Mathematical Finance

Exercise Sheet 11

Submit by 12:00 on Wednesday, December 11 via the course homepage.

Exercise 11.1 *(Equivalence of (NA))* Show that S satisfies (NA) if and only if 0 is maximal in $\mathcal{G}_{\mathrm{adm}}$.

Exercise 11.2 *(Discrete time: all elements are maximal)* Fix a finite time horizon $T \in \mathbb{N}$, and let $S = (S_k)_{k=0,1,\dots,T}$ be a discrete-time process. Let Θ denote the space of all predictable processes. Show that if S satisfies (NA), then neither $\mathcal{G}_{\mathrm{adm}}$ nor $G_T(\Theta)$ contain any non-maximal element.

Can you also show the result without using Theorem 1.2?

Exercise 11.3 *(Uniqueness)* Let S be an RCLL semimartingale satisfying (NFLVR) and let w be a feasible weight function. Suppose $f \in L^0$ with $|f| \leqslant w$, and assume $\alpha = \beta$ are finite, where as usual $\alpha := \inf \Gamma_+$ and $\beta := \sup \Gamma_-$ are the super- and subreplicating prices for f.

(a) Show that there exists a unique $g \in \mathcal{G}_w$ such that $f \leqslant \alpha + g$.

(b) Show directly (without using any results from the course) that g is maximal in $\mathcal{G}_w$.

Exercise 11.4 *(Maximality in a larger set)* Let S be an RCLL semimartingale satisfying (NFLVR). Let w be a feasible weight function and fix $g \in \mathcal{G}_w$. Define the random variable $w' := w + g^+$, where $g^+ := \max\{g, 0\}$. Show that w' is a feasible weight function, and that if g is maximal in $\mathcal{G}_w$ then g is also maximal in $\mathcal{G}_{w'}$.

Exercise 11.5 *(An example where $\Gamma_+ \cap \Gamma_-$ is large)* Construct a model for a financial market and a payoff f such that $\Gamma_+(f)$ and $\Gamma_-(f)$ intersect in more than one point.