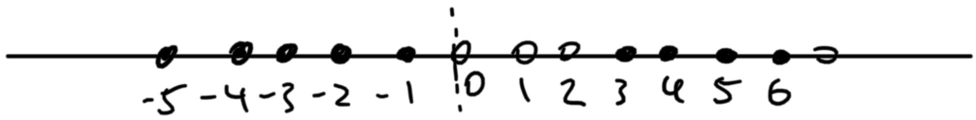
(1)



$$m = |S_+| - |S_-| = 4 - 1 = 3.$$

(2) $v_4 \wedge v_3 \wedge v_0 \wedge v_{-2} \wedge v_{-4} \wedge v_{-5} \wedge \cdots$

$$m = 2 - 2 = 0.$$

(4) (4.1) $s_i = i + m$ for $i \ll 0$, $s_i \geq i + m \ \forall i \leq 0$

$s_i \geq s_{i-1} + 1 \ \forall i \leq 0$

by the definition of semi-infinite monomials

$\lambda_i = s_{-i} + i - m \quad i \in \mathbb{Z}_{\geq 0}$

$\lambda_i = -i + i + m - m \ \forall i \gg 0.$
$= 0$

& $\lambda_i \geq -i + i + m - m = 0 \ \forall i \geq 0$

$\lambda_i - \lambda_{i+1} = s_{-i} + i - m - s_{-i-1} - i - 1 + m$

$\geq s_{-i-1} + 1 + i - m - s_{-i-1} - i - 1 + m$

$= 0$

(4.2)
(1) $\lambda_0 = 6 - 3 = 3 \qquad \lambda_4 = -1 + 4 - 3 = 0$

$\lambda_1 = 5+1-3=3$ $\vdots$

$\lambda_2 = 3$ $\Rightarrow \lambda = (3,3,3,3)$.

$\lambda_3 = 3$

(2) $\lambda_0 = 4$ $\lambda_3 = -2+3=1$

$\lambda_1 = 3+1=4$ $\lambda_4 = -4+4=0$

$\vdots$

$\lambda_2 = 0+2=2$

$\Rightarrow \lambda = (4,4,2,1)$.

(4.3) if $\lambda = \lambda'$ (enlarge λ, λ' by adding zeros $\lambda = (\lambda_0, \lambda_1, \dots \lambda_n, 0, 0, \dots)$).

then $s_{-i} = \lambda_i - i + m = \lambda'_i - i + m = s'_{-i}$

Similarly, if $S = S'$, then $\lambda = \lambda'$ by the above formula.

(4.4) We will prove the claim by induction on $|S_+|$.

Base case: $|S_+| = 0$, i.e., $S = \{0,-1,-2,-3,\dots\}$.

In this case, $\lambda = \phi$, $\lambda = (0,0,0,\dots)$.

$\Rightarrow |\lambda| = \sum s - \sum s = 0$.

$s \in S_+ \quad s \in S_-$

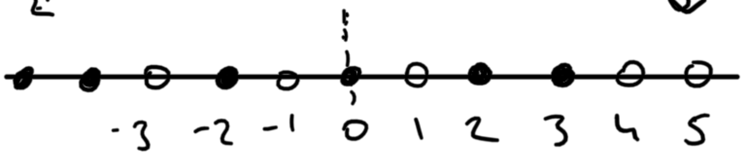
Assume the claim is true for all S s.t. $|S_+| = n$

we can get any S with $|S_+| = n+1$

by adding a black dot •

and removing a white dot o.

remove o   add •



More concretely, this corresponds to associating to a set S another set S', such that:

$$s'_0 = \hat{s}_0 \qquad s'_0 > s_0$$

$$s'_{-i} = s_{-i+1} \qquad \text{for } k \geq i \geq 1$$

$$s'_{-i} = s_{-i} \qquad \text{for } i \geq k+1$$

then
$$|\lambda'| = s'_0 + \sum_{k > i \geq 0} s_{-i} + (i+1) + \sum_{i > k} s_{-i} + i$$

$$= s'_0 + \sum_{k > i \geq 0} (s_{-i} + i) + \sum_{i > k} (s_{-i} + i) + k$$

$$= s'_0 + \sum_{k > i \geq 0} (s_{-i} + i) + \sum_{i \geq k} (s_{-i} + i) + k - s_{-k} - k$$

$$= s_0' + |\lambda| - s_{-k}$$

$$= s_0' + \sum_{s \in S_+} s - \sum_{s \in S_-} s - s_{-k}$$

$$= \sum_{s \in S'_+} s - \sum_{s \in S'_-} s \,,$$

i.e. we obtain the claim for

S', s.t. $|S'_+| = n+1$.

$\Rightarrow$ by induction the claim holds for all S.