Exercises 14.05.2025

Optional exercises for the lecture *Introduction to Floer homology* (401-3584-25L) Semester: Spring 2025 Lecturer: Dr. Jean-Philippe Chassé

Exercise 1. On $\mathbb{T}^2 = \mathbb{R}^2 / \mathbb{Z}^2$, consider the Hamiltonians

$$H(x, y) = \frac{1}{2\pi} \cos(2\pi x)$$
 and $G(x, y) = \frac{1}{2\pi} \sin(2\pi y)$.

- (1) Compute the composition $\varphi_H^t \circ \varphi_G^t$, and determine the time-dependent Hamiltonian $F : [0,1] \times \mathbb{T}^2 \to \mathbb{R}$ that generates this isotopy.
- (2) Determine the periodic orbits of $\{\varphi_F^t\}$, and identify which ones are contractible. Show that the contractible orbits are all nondegenerate, and compute their index.
- (3) Assuming that the pair (F, J₀) is regular, determine the Floer complex CF(F, J₀), and compute the resulting homology.

Hint: For the index computation, make use of Exercise 3 of 26.03.2025.