EXERCISES 05.03.2025

Optional exercises for the lecture *Introduction to Floer homology* (401-3584-25L)
Semester: Spring 2025
Lecturer: Dr. Jean-Philippe Chassé

Exercise 1. *Suppose for this exercise that M does not necessarily respect Assumptions 1 and 2. On the space $\mathscr{L}M$, consider the 1-form α_H defined via*

$$\alpha_H(Y) := \int_0^1 \omega_{x(t)}\left(\dot{x}(t) - X_H^t(x(t)), Y(t)\right)$$

for $Y \in \Gamma(x^*TM) = T_x\mathscr{L}M$.

Fix $x_0 \in \mathscr{L}M$ and a capping u_0 of x_0. Show that x_0 has a neighborhood U in $\mathscr{L}M$ and a function $a_H^U : U \to \mathbb{R}$ such that

$$a_H^U(x) = -\int_{\mathbb{D}} u^*\omega + \int_0^1 H_t(x(t))dt$$

for an appropriate capping u of $x \in U$. Conclude that α_H is closed.

Exercise 2. *Let $u : \mathbb{R} \times S^1 \to M$ be a solution to the Floer equation associated to $H : [0,1] \times M \to \mathbb{R}$ and $J \in \mathscr{J}_c(\omega)$. Suppose that $G : [0,1] \times M \to \mathbb{R}$ is 1-periodic, and define*

$$\widetilde{u}(s,t) := (\varphi_G^t)^{-1}(u(s,t)).$$

Find $\widetilde{H} : [0,1] \times M \to \mathbb{R}$ and $\widetilde{J} \in \mathscr{J}_c(\omega)$ such that $\widetilde{u}$ solves the Floer equation associated to $\widetilde{H}$ and $\widetilde{J}$.

Exercise 3. *Let $H : [0,1] \times M \to \mathbb{R}$ be a (possibly degenerate) Hamiltonian. Using the Arzelà–Ascoli theorem, show that $\mathrm{Crit}(\mathscr{A}_H)$ is compact in $\mathscr{L}M$ (in any C^k topology for $0 \le k \le \infty$).*