Exercises 09.04.2025

Optional exercises for the lecture *Introduction to Floer homology* (401-3584-25L)
Semester: Spring 2025
Lecturer: Dr. Jean-Philippe Chassé

Exercise 1. *Let $L : E \longrightarrow F$ be a Fredholm operator. Show that the image of L is closed.*

Exercise 2 (4.6.5)**.** *Let E, F, and G be Banach spaces, and let $L : E \longrightarrow G$ and $L' : F \longrightarrow G$ be continuous and linear. Suppose that L is Fredholm and $L + L' : E \oplus F \longrightarrow G$ is surjective. Show that $L + L'$ admits a continuous right inverse.*

Exercise 3 (4.7.1)**.** *Let $u : \mathbb{R} \times S^1 \longrightarrow M$ be a solution of the Floer equation for J and H, and see it as a function $\mathbb{R}^2 \longrightarrow M$ that is 1-periodic in the second variable. Show that $v(s,t) := (\varphi_H^t)^{-1}(u(s,t))$ solves*

$$\begin{cases} \frac{\partial v}{\partial s} + \tilde{J}_t(v)\frac{\partial v}{\partial t} = 0 \\ v(s, t+1) = \varphi_H^1(v(s,t)) \end{cases},$$

where $\tilde{J}_t := (\varphi_H^t)^* J$.

Exercise 4 (Proposition 4.8.3)**.** *Let E, F, and G be Banach spaces, and let $L : E \longrightarrow F$ and $K : E \longrightarrow G$ be linear and continuous. Suppose that K is compact, i.e. it sends compact sets to bounded ones, and that there exists $C > 0$ such that*

$$||x||_E \leq C\left(||Lx||_F + ||Kx||_G\right)$$

for all $x \in E$. Show that the kernel of L is finite dimensional.

Hint: *The closed unit ball in a Banach space is compact if and only if said space is finite dimensional.*