ETH Zürich, FS 2025
D-MATH
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Applied Stochastic Processes

Mock Exam

Quiz Mock Exam 1

(a) Let $(N_t)_{t\geq 0}$ and $(M_t)_{t\geq 0}$ be two independent Poisson processes with intensity 1. Which of the following are true?

1. $(N_t + M_t)_{t\geq 0}$ is a Poisson process with intensity 2.
2. Let $X_t = N_{2t}$ for all $t \geq 0$. Then $(X_t)_{t\geq 0}$ is a Poisson process with intensity 2.
3. $N_t \xrightarrow{a.s.} \infty$ as $t \to \infty$.

Exercise Mock Exam 2 Let $Z_1, Z_2, \ldots$ be iid random variables uniformly distributed in $\{1,2,3,4\}$ (i.e., $\mathbb{P}(Z_n = i) = \frac{1}{4}$ for every n and every $i \in \{1,2,3,4\}$). This corresponds to independent rolls of a 4-faced die. We consider the sum of the outcomes modulo 3: For every $n \geq 0$, we set

$$X_n = (Z_1 + \cdots + Z_n) \mod 3,$$

where $(k \mod 3) \in \{0,1,2\}$ denotes the remainder of an integer k by the division by 3. By convention, we have $X_0 = 0$. The goal of the exercise is to show that

$$\lim_{n\to\infty} \mathbb{P}(X_n = 0) = \frac{1}{3}. \tag{1}$$

(a) Compute $\mathbb{P}(X_1 = 0)$ and $\mathbb{P}(X_2 = 0)$.

(b) Prove that $(X_n)_{n\geq 0}$ is a Markov Chain on $S = \{0,1,2\}$ with initial distribution δ_0 and transition probability P given by

$$p_{i,j} = \begin{cases} \frac{1}{4} & \text{if } j = i \text{ or } j = (i+2) \mod 3, \\ \frac{1}{2} & \text{if } j = (i+1) \mod 3. \end{cases}$$

(c) Prove that P is irreducible and aperiodic.

(d) Prove Equation (1).

(e) Does the convergence in Equation (1) still hold if we replace the 4-face die with a standard cubic 6-face die?