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Introduction to Mathematical Finance

Exercise sheet 1

Please submit your solutions online until Tuesday 22:00, 04/03/2025.

Exercise 1.1

(a) Construct a market with arbitrage of the first kind but with no arbitrage of the second kind.

(b) Construct a market with arbitrage of the second kind but with no arbitrage of the first kind.

(c) Prove Proposition I.3.1. That is suppose there exists an asset $\mathcal{D}^l$ with $\mathcal{D}^l \geq 0$ and $\mathcal{D}^l \not\equiv 0$. Show that under this assumption, the market is arbitrage-free iff there is no arbitrage of first kind.

Exercise 1.2 Let $\mathcal{C} := \mathbb{R} \times \mathbb{R}^K$ be the consumption space with the payoff matrix $\mathcal{D}$ and let e^i, π be an endowment, and a price vector, respectively. Recall the budget set

$$B(e^i, \pi) := \{c \in \mathcal{C} : \exists \vartheta \in \mathbb{R}^N \text{ with } c_0 \leq e^i_0 - \vartheta \cdot \pi \text{ and } c_T \leq e^i_T + \mathcal{D}\vartheta\}.$$

(a) Show $c \in B(e^i, \pi) \iff c - e^i \in B(0, \pi) \Longleftarrow c - e^i$ is attainable with 0 initial wealth.

(b) Show by an example that the converse of the second implication is not true in general.

Exercise 1.3 Suppose $\mathcal{D}$ is complete. Show that $B(e, \pi) = \mathcal{C}$ for all e if and only if there exists arbitrage of the second kind.