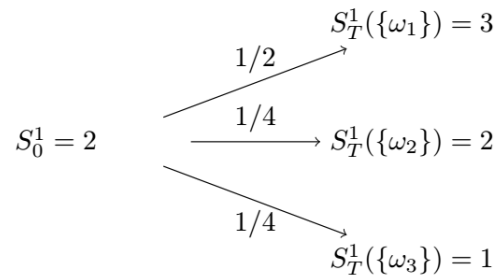


Introduction to Mathematical Finance

Exercise sheet 5

Exercise 5.1 Consider a trinomial two-asset model. The first asset is a risk-free bond with initial value $S_0^0 = 1$ and the second asset is a risky stock with initial value $S_0^1 = 2$ and whose evolution under the real world measure P is given by the following tree:



We also suppose that the spot interest rate is $r = 0$.

- (a) Find all risk-neutral measures for this model.

Now introduce a call option on the risky asset with strike $K = 2$ and maturity T .

- (b) What is the terminal payoff H of this contingent claim?
- (c) Find the least expensive super replicating portfolio, i.e. the portfolio that attains the infimum in the definition of $\pi_s(H)$.
- (d) Find the most expensive sub-replicating portfolio.

Exercise 5.2

Consider a model with $d = 1$ traded risky asset X with $X_0 = 1$ and

$$\Delta X_k = \eta_k, \quad k = 1, 2, 3,$$

where the η_k are i.i.d. $\eta_1 \sim \mathcal{N}(0, 1)$ -distributed.

- (a) Suppose that a trader decides at time $k = 0$ to buy 2 shares, to sell 3 shares at $k = 1$ and then to buy 1 share at time $k = 2$. Denote by G_k his cumulative gain from the corresponding self-financing trading strategy. Find the distribution of G_3 .

- (b) Suppose that $\mathcal{F}_k = \sigma(X_1, \dots, X_k)$ for $k = 1, 2, 3$. Show that there is no arbitrage in this model.

Exercise 5.3 Consider the standard model for a financial market in finite discrete time with a numéraire S^0 .

- (a) Show that a strategy ψ is self-financing for S if and only if it is self-financing for S/S^0 .
- (b) Show that S satisfies NA' if and only if S/S^0 satisfies NA'.
- (c) Show that S satisfies NA if and only if S/S^0 satisfies NA.